Finite frequency response of small magnetic structures under an external static field

S. T. Chui,[1] V. Novosad,[2] and S. D. Bader[2]
[1]*Bartol Research Institute and Department of Physics and Astronomy, University of Delaware, Newark, Delaware 19716, USA*
[2]*Materials Science Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
(Received 10 March 2009; revised manuscript received 3 August 2009; published 31 August 2009)

We apply the Holstein-Primakoff and Bogoliubov transformations to compute the spin-wave states of small magnetic structures including the effect of the dipolar interaction. We found that as the film gets thicker, states with a significant $q=0$ component, are hybridized with states with higher Fourier components. In the presence of a static magnetic field opposite to the magnetization direction, surface states that are responsible for magnetization reversal are coupled to the extended states. The response function is increased by an order of magnitude. This suggests an intriguing scenario for assisted switching of the magnetization with an additional external ac field.

DOI: 10.1103/PhysRevB.80.054419 PACS number(s): 75.75.+a, 64.10.+h, 75.30.Ds

I. INTRODUCTION

Motivated by recent studies on spin-polarized transport, magnetic memory, sensor, and spin wave logic devices using small structures, there have been burgeoning interest in the physics of spin waves in these systems.[1–4]

Of interest is the response of a small structure in the presence of both external static and time varying fields in different directions; such as occurs in the study of the ferromagnetic resonance and, more recently, in the context of assisted switching with ac fields.[5,6] A major obstacle in the large scale commercialization of magnetic random access memory is the precise control of the switching of the magnetization. Assisted switching offers one possible way to address this issue. The external fields are usually relatively uniform spatially and thus couples strongly only to excitations with a significant $q=0$ Fourier component. Because the structure is finite and may not be of uniform shape, states of different q values are coupled; more than one excitation can exhibit a significant $q=0$ response.

Most recent numerical calculations of the spin wave spectrum are obtained by solving for the time evolution of the spins under the Landau-Lifshitz-Gilbert (LLG) equation.[2] This approach is quite time consuming. Here, we directly compute the spin-wave spectrum from the diagonalization of its Hamiltonian matrix containing the spin-wave operators. This approach is much faster and enables us to investigate how the properties change as the system parameters are changed. Grimsditch and co-workers[1] have studied the spin-wave spectrum by diagonaling the dynamical matrix obtained from linearizing the classical phenomenological LLG equation. They have not discussed the response function.

We found dramatic changes as the system parameters are changed. We consider elliptical elements in the XY plane with the magnetization along the X axis. In Fig. 1, we show the amplitude of the Y component of the spin for "extended" spin-wave states with the largest $q=0$ Fourier components with increasing dipolar interaction. When the dipolar interaction is zero (left), the state is nearly uniform. Recent simulations on different device prospects were carried out on very thin films. For a film of thickness 20 Å, (middle) the state is no longer uniform. When a film thickness is 200 Å, a much higher q Fourier component is present. At the same time, more than one state can exhibit a significant $q=0$ response.

We studied the response to a uniform but time varying external field in a direction perpendicular to the magnetization in the presence of a static magnetic field applied along the magnetization axis; as it is typical in ferromagnetic resonance experiments. When the static field is opposite to the magnetization direction, the peak of the finite frequency response function can increase by an order of magnitude. This suggests an intriguing scenario for assisted switching of the magnetization with an additional external ac field. We now describe our results in detail.

II. FORMULATION

The usual spin-wave calculation assumes translational symmetry and is diagonalized assuming that the crystal momentum q is a good quantum number. We first generalize this to a finite system without translational symmetry.

We write the Hamiltonian of the system as $U=\Sigma_{a,b=X,Y,Z;i,j}M^0_{ab}(ij)S_{ai}S_{bj}$. For a finite system, the spins in equilibrium are no longer completely parallel to each other, the spin at site n is aligned along a direction $\mathbf{S}^0(n)$. We call this the local x direction and look for a small deviation of the spin from this equilibrium direction. We follow the usual Holstein-Primakoff transformation and introduce the canonical variables:

$$S_z(n) = (S/2)^{1/2}[a(n) + a^*(n)]$$

$$S_y(n) = i(S/2)^{1/2}[a(n) - a^*(n)]$$

$$S_x(n) = S - a^*(n)a(n). \tag{1}$$

With this, the commutation relationship between the components of the spin operator is preserved. The local coordinate system with lower case subscripts is related to the space-fixed system with upper case subscripts by a rotation matrix $R_{iJ}(n)$ so that $S_i(n)=\Sigma_A R_{iA}(n)S_A(n)$. For example, if the spins lie in the plane in equilibrium (the XY plane) so that $\mathbf{S}^0(n)=[S^0_X(n),S^0_Y(n),0]$, then $R_{xX}(n)=R_{yY}(n)=S^0_X(n)$, $R_{xY}(n)=-R_{yX}(n)=S^0_Y(n)$. Ignoring the constant terms, the Hamiltonian has the form:

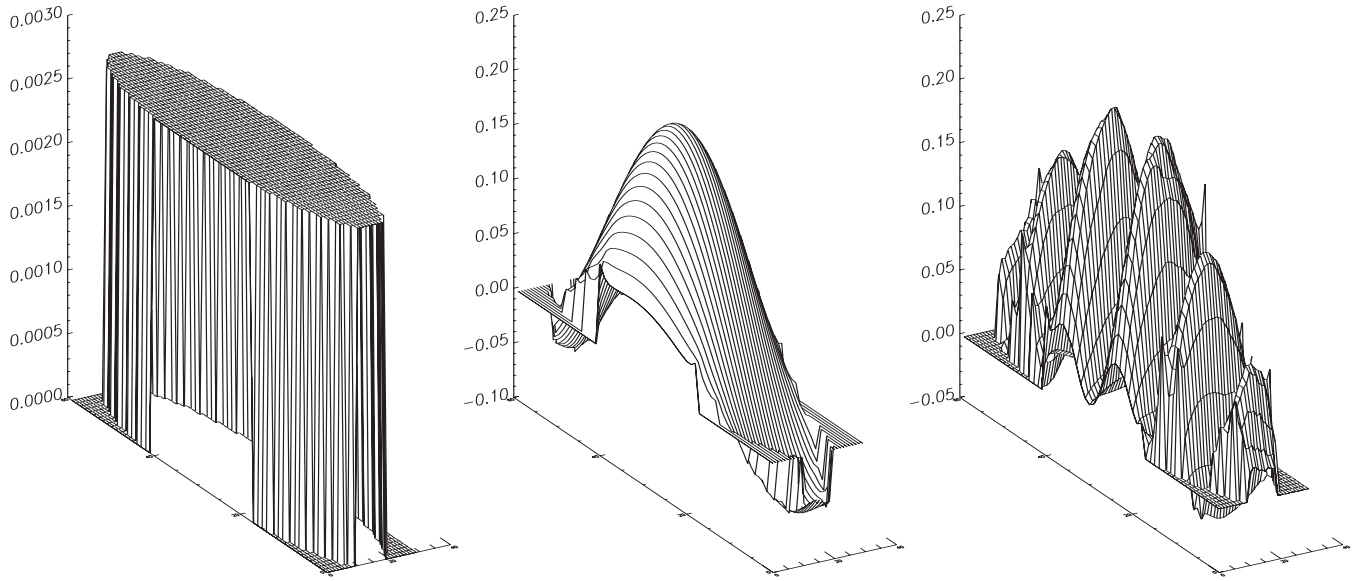

FIG. 1. The amplitude of the Y component of the spins for the eigenstate with the largest $q=0$ component at different sites for the case with no dipolar interaction (left), for a 20 Å thick permalloy film (middle), and for a 200 Å thick permalloy film of elliptical shape with dimensions 1×0.5 microns. The short-range exchange interaction remains unchanged.

$$U = \sum_{i,j} [M_z(ij) - M_y(ij)](S/2)[a(i)a(j) + a^*(i)a^*(j)] + [M_z(ij) + M_y(ij) - 2h_{xi}\delta_{i,j}]Sa^*(i)a(j), \quad (2)$$

where we sum over the positions labeled by i, j. $M_a(ij) = \Sigma_{AB} R_{aA}(i) R_{aB}(j) M^0_{AB}(ij)$, and $h_{xi} = \Sigma_j M_x(ij)$. Because we have expanded from the equilibrium configuration, there are no terms linear in the operators. We next introduce the "Bogoliubov" transformation:

$$b = \sum_i u(i)a(i) + v(i)a^*(i)$$

$$b^* = \sum_i u(i)^* a^*(i) + v^*(i)a(i) \quad (3)$$

$$1 = \sum_i |u(i)|^2 - |v(i)|^2 \quad (4)$$

and diagonalize the Hamiltonian with the equation of motion method. The time rate of change of the operator b is given by $[U,b] = \Sigma u(i)[U,a(i)] + v(i)[U,a^*(i)]$, which in turn has to be equal to $-\omega b$. (We use units so that $\hbar = 1$.) We get the eigenvalue equations

$$(M_z^T - M_y^T)u - (M_z^T + M_y^T - 2h_x\delta_{i,j})v = \omega v,$$

$$-(M_z^T - M_y^T)v + (M_z^T + M_y^T - 2h_x\delta_{i,j})u = \omega u. \quad (5)$$

This can also be written as $G_z(u-v) = \omega(u+v)$, and $G_y(u+v) = \omega(u-v)$, where $G_a = 2(M_a^T - h_x\delta_{i,j})$ for $a = z, y$. Simplifying this we arrive at the equation:

$$G_z G_y (u+v) = \omega^2 (u+v). \quad (6)$$

The eigen problem involves the product of two matricies. Even though the product matrix $G_z G_y$ is not symmetric, as shown below, the matricies M and hence the individual matricies $G_{y,z}$ are real and symmetric. There are subroutines in LAPACK that deal with this type of eigenvalue problem if one of the matricies is positive definite. For our specific problem of thin films, the matrix G_z satisfies this condition because of the shape anisotropy gap that confines the spins in the xy plane. Since $(u-v) = \omega G_z^{-1}(u+v)$, the normalization condition (4), which comes from the quantization condition of the spin wave $[b, b^+] = 1$, becomes

$$(u+v)^T G_z^{-1}(u+v) = 1/\omega. \quad (7)$$

Here, $1/\omega$ means a diagonal matrix with diagonal elements $(1/\omega_1, 1/\omega_2, \ldots)$ The LAPACK routines uses the normalization condition $(u+v)^T G_z^{-1}(u+v) = 1$. This can be easily modified. The susceptibility along direction I, χ_I, is calculated as usual from linear response theory[7] as a spin autocorrelation function:

$$\chi_I(q,\omega) = (1/N)\sum_a |\langle a|S_I(q)|0\rangle|^2 \omega_a / [\omega^2 - \omega_a^2 + i\omega\omega_a\alpha], \quad (8)$$

where $S_I(q) = \Sigma_j S_I(j) \exp(iq \cdot R_j)$. α is a phenomenological damping constant. In this paper, we shall focus on $I = y$, corresponding to a uniform time varying external field in the plane in a direction perpendicular to the magnetization. We next relate $M_{y,z}$ and h_x to the physical quantities of the system.

III. IMPLEMENTATION

In this paper, we assume that the film thickness is much less than the magnetic length so that all spins perpendicular to the film are parallel to each other. Let us consider the spins in a plane with the magnetization along the X axis and

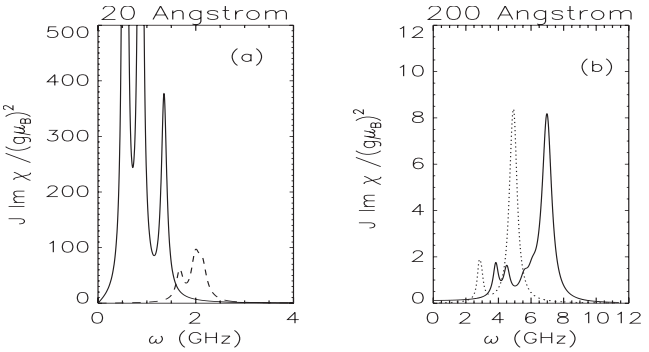

FIG. 2. The imaginary part of the response function $J\chi_Y(q=0,\omega)/(g\mu_B)^2$ for permalloy films of elliptical shapes with dimensions 1×0.5 microns. Here, J is the exchange, μ_B is the Bohr magneton, g is the g factor. α was set to a value of 0.1. (a) For a thickness of 20 Å at zero (dashed line) and a reversing field H_x of −30 Oe (solid line). (b) For a 200 Å thick film at zero (dashed line) and a field of 300 Oe (solid line).

uniaxial anisotropy along the X direction. The Hamiltonian is given by:

$$U = -1/2\sum_{\langle i,j\rangle} J_{ij}\mathbf{S}(i)\cdot\mathbf{S}(j) - K/2\sum_i S_x^2(i) - H\sum_i S_x(i) + 1/2g\sum_{ij}\left[\sum_{\alpha,\beta=x,y} D_{\alpha\beta}(R_{ij})S_\alpha(i)S_\beta(j) - [D_{xx}(R_{ij}) + D_{yy}(R_{ij})]S_z(i)S_z(j)\right], \quad (9)$$

where

$$D_{\alpha\beta}(R) = \delta_{\alpha\beta}/R^3 - 3R_\alpha R_\beta/R^5 \quad (10)$$

and J, g, and K are the exchange, dipolar, and intrinsic anisotropy interaction constants, respectively. For permalloy films we have used the "bare" interaction $J=450$ K, $g=0.055$ K, and $K=4.3\times10^{-4}$ K.

The Fourier transform of $D_{\alpha\beta}(R)$ has the form:

$$d_{\alpha\beta}(q) = \sum_R [cos(qR) - 1]D_{\alpha\beta}(R) + d^0_{\alpha\beta} = D_{\alpha\beta}(q) + d^0_{\alpha\beta}, \quad (11)$$

where $d^0_{\alpha\beta}=\Sigma_R[\delta_{\alpha\beta}/R^3-3R_\alpha R_\beta/R^5]$, $D_{\alpha\beta}(q)$ is the Fourier transform of the dipolar interaction. For an infinite system it is identical in form to the dynamical matrix of the two-dimensional (2D) electron crystal and has been evaluated previously,[8,9] $D_{\alpha\beta}(q)\to 2\pi q_\alpha q_\beta/qv_0+O(q^2)$ for $q\to0$, v_0 is the unit cell volume of the crystal.

We thus get, for $a,b=x,y$

$$2M^0_{ab}(ij) = H_a\delta_{a,b} - \delta_{\langle i,j\rangle}J_{ij}\delta_{a,b} + gD_{ab}(R_{ij}) - \delta_{a=x=b}\delta_{i,j}K/2 \quad (12)$$

$$2M^0_{zz}(ij) = -\delta_{\langle ij\rangle}J_{ij} - g[D_{xx}(R_{ij}) + D_{yy}(R_{ij})] \quad (13)$$

Just as in the study of dynamics, we shall approximate the system by a collection of block spins with renormalized interactions between them.[10] This approximation excludes from our consideration the high-lying states with Fourier components that are larger than the inverse block spin length. The equilibrium spin configuration is obtained from a finite temperature Monte Carlo simulation using a code we have developed and optimized over the past ten years.[9]

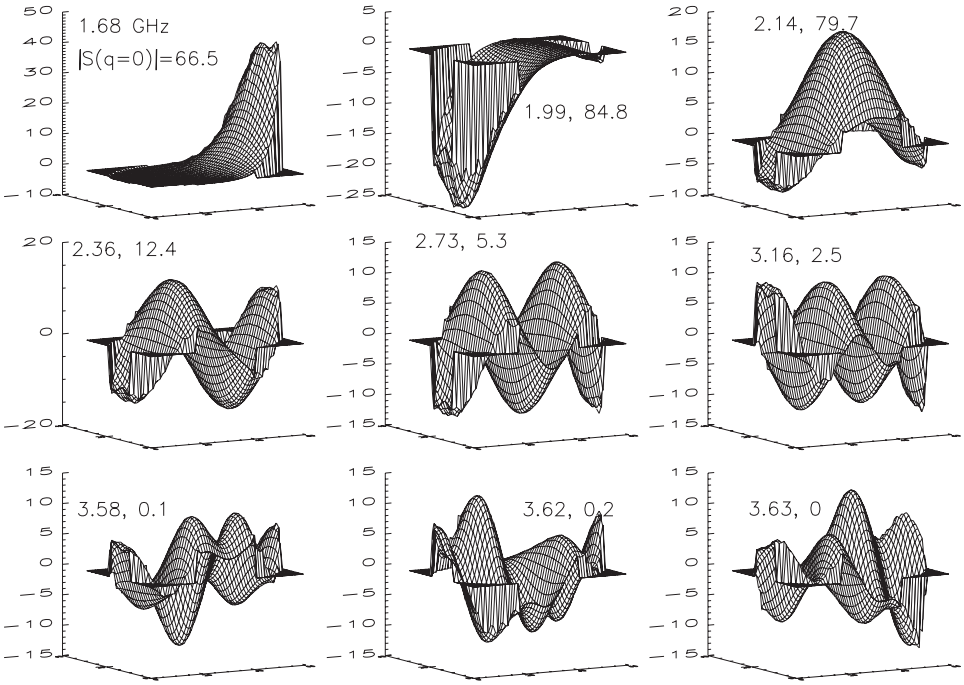

FIG. 3. The amplitude of the Y component of the spins at different sites for the first nine eigenstates in a 20 Å thick permalloy film of elliptical shape with dimensions 1×0.5 microns at zero external field. The pair of numbers shows the spin wave frequency in GHz and the magnitude of the structure factor.

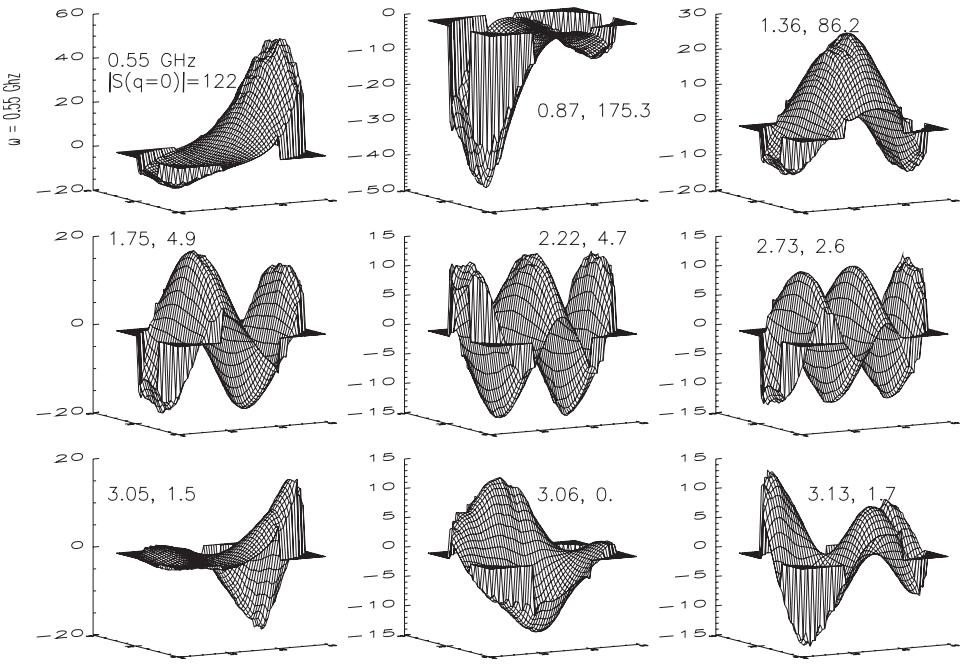

FIG. 4. The amplitude of the Y component of the spins at different sites for the first nine eigenstates in a 20 Å thick permalloy film of elliptical shape with dimensions 1×0.5 microns. A static field $H_x=-30$ Oe is applied. The spin-wave frequency in GHz and the magnitude of the structure is also shown.

IV. RESULTS

To illustrate the physics, unless otherwise stated we focus in this paper on elliptical structures made of permalloy with dimensions 1×0.5 microns and of different thicknesses. We have calculated the eigenstates in the presence of static magnetic fields applied along the magnetization direction (x) and obtained the imaginary response functions as a function of frequency. In Fig. 2(a), we show results for a 20 Å permalloy film at zero external field (dashed line) and at a finite reversing field of −30 Oe (solid line.) More than one peak is seen in the response function. This is because more than one state can exhibit a significant $q=0$ response. As the reversing field becomes nonzero, the positions of the peaks decreases (solid line in Fig. 2).

No such increase is observed for a forward field, as is illustrated in Fig. 2(b) for a 200 Å permalloy film.

In Fig. 3, we show the first nine spin-wave states (from left to right and top to bottom) for a 20 Å film at zero field. The spin-wave frequency in GHz and the magnitude of the structure factor is also shown for each state. The first two are surface states localized near the two edges. A large spectral weight for the $q=0$ response comes from the surface states. These modes basically correspond to the movement of edge domain walls, and are responsible for magnetization reversal under a *static* reversing field. For a rectangular shape sample, there are more surface states, corresponding to higher spatial Fourier components along the y directions, similar to previous results.[1,2]

As the reversing field becomes nonzero, the spin wave frequency decreases (solid line of Fig. 2) and their contribution to $|S_Y(q=0)|$ increases. The corresponding spin-wave states are shown in Fig. 4. At a finite reversing field, the surface states are lower in energy, become more extended and hybridized more strongly with the extended states that coupled to a *uniform* external field. These states are responsible for the large susceptibility in Fig. 2(a).

For spin waves of wave vector q in an *infinite* sample, the spin-wave energy is given by $\omega=[G_z(q)G_y(q)]^{1/2}$, where in the long wavelength limit $G_z(q)=\Delta_z+\frac{Z}{2}JS(a_0q)^2-gS\frac{2\pi qa_0}{v_0}$, $G_y(q)=\Delta_y+\frac{Z}{2}JS(a_0q)^2+gS\frac{2\pi q_y^2}{v_0q}$, $\Delta_y=H+K+g(d_{xx}^0-d_{yy}^0)$, $\Delta_z=K-g(d_{yy}^0+2d_{xx}^0)$, a_0 is the lattice constant. Z is the number of nearest neighbors. Because of the dipolar term, the minimum of ω occurs not at $q_2=0$ but at a *finite* $q_y=0.5g\Delta_x/[J(\Delta_x+\Delta_z)]$, which we call q_1. This dispersion is illustrated in Fig. 5 as the dotted lines. For a structure of

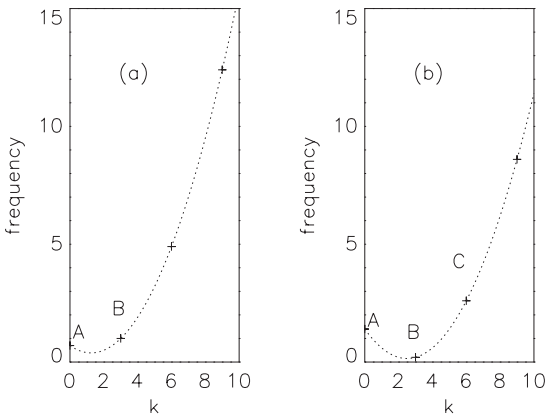

FIG. 5. Illustrative magnon dispersion for an infinite system in arbitrary units (dotted lines) for a thin (a) and a thick sample (b). The units are arbitrary. The allowed wave vectors are shown by the solid crosses.

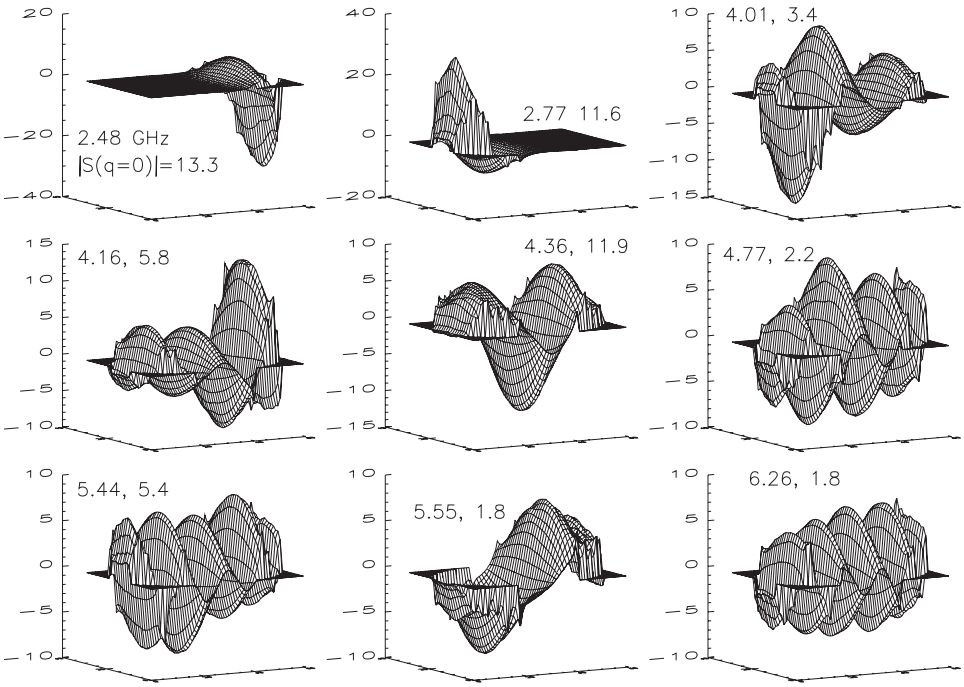

FIG. 6. The Y component of the spins at different sites for the first nine eigenstates in a 200 Å thick permalloy film of elliptical shape with dimensions 1×0.5 microns at zero external field. The spin-wave frequency in GHz and the magnitude of the structure is also shown.

length L_x, the possible wave vectors are integer multiples of $G=2\pi/L_x$. This is illustrated in Fig. 5(a) as the solid crosses. Thus for small structures so that $G>q_1$, the lowest extended excitation can still occur at the wave vector $q_2=0$, as is illustrated in Fig. 5(a). This corresponds to what we observe for the 20 Å films shown in Fig. 3. As the film gets thicker, the effective dipolar coupling is increased and q_1 decreases; the lowest excitation now occurs at a finite wave vector. This state is illustrated in Fig. 5(b) labeled as B. This corresponds to what we observe for the 200 Å films shown in Fig. 6. As we see in Fig. 5, for the thin film case (a), the $q=0$ state hybridizes with the state labeled B because they are close in energy. For the thick film case (b), the $q=0$ state now hybridizes also with the state labeled C, which is also close in energy but possesses a larger q. This corresponds to what we showed in Fig. 1.

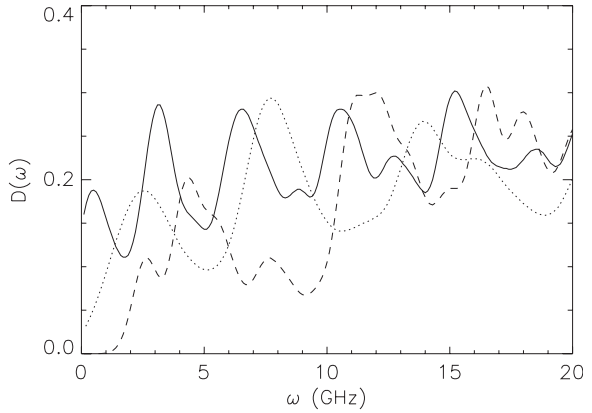

FIG. 7. The magnon density of states for different structures. [dotted and dashed lines: 20 and 200 Å thick films at zero field; solid line: 20 Å thick film in a finite reversing field of 30 G. A Gaussian width of 0.9 GHz is assumed for each state.

The hybridization of states of different wave vectors for thicker films is illustrated in Fig. 6 where we show the lowest 9 spin wave states for a 200 Å thick permalloy film. As can be seen, the states are now different from those in Fig. 3.

The magnon densities of states are shown in Fig. 7. Under a finite reversing field (solid line), the density of states at low energy increases, which may enhance the magnitude of the 1/f noise in these structures. This comes about because the spin wave energy is now decreased. The finite value of the density of states at zero energy comes from a phenomenological lifetime of 0.9 GHz that we have used in calculating the density of states. As the film gets thicker (200 Å) the energy of excitation increases (dashed line).

The effects discussed in this paper are primarily due to the presence of the dipolar interaction and not due to the finite size of the system. When the dipolar interaction is absent, no surface state is found. There is only one state with a significant $q=0$ component, which is nearly uniform in space. The ordering of the states is what one expects.

In conclusion, in this paper, we described a way to compute the spin-wave excitation for small magnetic structures numerically. The dipolar interaction is found to cause many interesting effects. We focus on the hybridization of states with different spatial Fourier components. We found that as the film gets thicker, states with a significant $q=0$ component are hybridized with states with higher Fourier components. In the presence of a static magnetic field opposite to the magnetization direction, surface states that are responsible for magnetization reversal are coupled to the extended states. The response function can increase by an order of magnitude. This may be of interest in assisted switching by an ac field.

ACKNOWLEDGMENTS

This work was supported by the U.S. Department of Energy, Office of Basic Energy Sciences under Contracts No. DE-AC02-06CH11357 (at Argonne) and No. DE-FG02-07ER46360 (at UD).

[1] M. Grimsditch, L. Giovannini, F. Montoncello, F. Nizzoli, Gary K. Leaf, and Hans G. Kaper, Phys. Rev. B **70**, 054409 (2004).
[2] G. Gubbiotti, M. Madami, S. Tacchi, G. Carlotti, A. O. Adeyeye, S. Goolaup, N. Singh, and A. N. Slavin, J. Magn. Magn. Mater. **316**, e338 (2007).
[3] K. Y. Guslienko, R. W. Chantrell, and A. N. Slavin, Phys. Rev. B **68**, 024422 (2003).
[4] J. P. Park, P. Eames, D. M. Engebretson, J. Berezovsky, and P. A. Crowell, Phys. Rev. Lett. **89**, 277201 (2002).
[5] G. Bertotti, C. Serpico, and I. D. Mayergoyz, Phys. Rev. Lett. **86**, 724 (2001).
[6] Z. Z. Sun and X. R. Wang, Phys. Rev. Lett. **97**, 077205 (2006).
[7] For an example, see G. D. Mahan, *Many Particle Physics* (Plenum Press, New York, 1981).
[8] L. Bonsall and A. Maradudin, Phys. Rev. B **15**, 1959 (1977).
[9] S. T. Chui, in *Magnetic Hysteresis in Novel Magnetic Materials*, edited by G. Hadjipanayis, NATO ASI Series E (Kluwer, Dordrecht, 1997), Vol. 338.
[10] For films of thickness d, we have used block spins of dimensions 166 Å $\times$ 166 Å $\times d$. This transverse dimension is much less than the magnetic length of Permalloy.